Section 4.6 Exponential and Logarithmic Models

While we have explored some basic applications of exponential and logarithmic functions, in this section we explore some important applications in more depth.

Radioactive Decay

In an earlier section, we discussed radioactive decay – the idea that radioactive isotopes change over time. One of the common terms associated with radioactive decay is half-life.

Half Life

The **half-life** of a radioactive isotope is the time it takes for half the substance to decay.

Given the basic exponential growth/decay equation $h(t) = ab^t$, half-life can be found by solving for when half the original amount remains; by solving $\frac{1}{2}a = a(b)^t$, or more simply $\frac{1}{2} = b^t$. Notice how the initial amount is irrelevant when solving for half-life.

Example 1

Bismuth-210 is an isotope that decays by about 13% each day. What is the half-life of Bismuth-210?

We were not given a starting quantity, so we could either make up a value or use an unknown constant to represent the starting amount. To show that starting quantity does not affect the result, let us denote the initial quantity by the constant *a*. Then the decay of Bismuth-210 can be described by the equation $Q(d) = a(0.87)^d$.

To find the half-life, we want to determine when the remaining quantity is half the original: $\frac{1}{2}a$. Solving,

$\frac{1}{2}a = a(0.87)^d$	Dividing by *a*,
$\frac{1}{2} = 0.87^d$	Take the log of both sides
$\log\left(\frac{1}{2}\right) = \log\left(0.87^d\right)$	Use the exponent property of logs
$\log\left(\frac{1}{2}\right) = d\log\left(0.87\right)$	Divide to solve for *d*

$$d = \frac{\log\left(\frac{1}{2}\right)}{\log(0.87)} \approx 4.977 \text{ days}$$

This tells us that the half-life of Bismuth-210 is approximately 5 days.

Example 2

Cesium-137 has a half-life of about 30 years. If you begin with 200mg of cesium-137, how much will remain after 30 years? 60 years? 90 years?

Since the half-life is 30 years, after 30 years, half the original amount, 100mg, will remain.

After 60 years, another 30 years have passed, so during that second 30 years, another half of the substance will decay, leaving 50mg.

After 90 years, another 30 years have passed, so another half of the substance will decay, leaving 25mg.

Example 3

Cesium-137 has a half-life of about 30 years. Find the annual decay rate.

Since we are looking for an annual decay rate, we will use an equation of the form $Q(t) = a(1+r)^t$. We know that after 30 years, half the original amount will remain. Using this information

$\frac{1}{2}a = a(1+r)^{30}$ Dividing by a

$\frac{1}{2} = (1+r)^{30}$ Taking the 30th root of both sides

$\sqrt[30]{\frac{1}{2}} = 1+r$ Subtracting one from both sides,

$$r = \sqrt[30]{\frac{1}{2}} - 1 \approx -0.02284$$

This tells us cesium-137 is decaying at an annual rate of 2.284% per year.

Try it Now

Chlorine-36 is eliminated from the body with a biological half-life of 10 days[3]. Find the daily decay rate.

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[3] http://www.ead.anl.gov/pub/doc/chlorine.pdf

Example 4

Carbon-14 is a radioactive isotope that is present in organic materials, and is commonly used for dating historical artifacts. Carbon-14 has a half-life of 5730 years. If a bone fragment is found that contains 20% of its original carbon-14, how old is the bone?

To find how old the bone is, we first will need to find an equation for the decay of the carbon-14. We could either use a continuous or annual decay formula, but opt to use the continuous decay formula since it is more common in scientific texts. The half life tells us that after 5730 years, half the original substance remains. Solving for the rate,

$\frac{1}{2}a = ae^{r5730}$ Dividing by a

$\frac{1}{2} = e^{r5730}$ Taking the natural log of both sides

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{r5730}\right)$ Use the inverse property of logs on the right side

$\ln\left(\frac{1}{2}\right) = 5730r$ Divide by 5730

$$r = \frac{\ln\left(\frac{1}{2}\right)}{5730} \approx -0.000121$$

Now we know the decay will follow the equation $Q(t) = ae^{-0.000121t}$. To find how old the bone fragment is that contains 20% of the original amount, we solve for t so that $Q(t) = 0.20a$.

$$0.20a = ae^{-0.000121t}$$
$$0.20 = e^{-0.000121t}$$
$$\ln(0.20) = \ln\left(e^{-0.000121t}\right)$$
$$\ln(0.20) = -0.000121t$$
$$t = \frac{\ln(0.20)}{-0.000121} \approx 13301 \text{ years}$$

The bone fragment is about 13,300 years old.

Try it Now

2. In Example 2, we learned that Cesium-137 has a half-life of about 30 years. If you begin with 200mg of cesium-137, will it take more or less than 230 years until only 1 milligram remains?

Doubling Time
For decaying quantities, we asked how long it takes for half the substance to decay. For growing quantities we might ask how long it takes for the quantity to double.

Doubling Time

The **doubling time** of a growing quantity is the time it takes for the quantity to double.

Given the basic exponential growth equation $h(t) = ab^t$, doubling time can be found by solving for when the original quantity has doubled; by solving $2a = a(b)^x$, or more simply $2 = b^x$. Again notice how the initial amount is irrelevant when solving for doubling time.

Example 5

Cancer cells sometimes increase exponentially. If a cancerous growth contained 300 cells last month and 360 cells this month, how long will it take for the number of cancer cells to double?

Defining *t* to be time in months, with *t* = 0 corresponding to this month, we are given two pieces of data: this month, (0, 360), and last month, (-1, 300).

From this data, we can find an equation for the growth. Using the form $C(t) = ab^t$, we know immediately *a* = 360, giving $C(t) = 360b^t$. Substituting in (-1, 300),

$$300 = 360b^{-1}$$

$$300 = \frac{360}{b}$$

$$b = \frac{360}{300} = 1.2$$

This gives us the equation $C(t) = 360(1.2)^t$

To find the doubling time, we look for the time until we have twice the original amount, so when *C(t)* = 2*a*.

$$2a = a(1.2)^t$$

$$2 = (1.2)^t$$

$$\log(2) = \log(1.2^t)$$

$$\log(2) = t\log(1.2)$$

$$t = \frac{\log(2)}{\log(1.2)} \approx 3.802 \text{ months.}$$

It will take about 3.8 months for the number of cancer cells to double.

Example 6

Use of a new social networking website has been growing exponentially, with the number of new members doubling every 5 months. If the site currently has 120,000 users and this trend continues, how many users will the site have in 1 year?

We can use the doubling time to find a function that models the number of site users, and then use that equation to answer the question. While we could use an arbitrary *a* as we have before for the initial amount, in this case, we know the initial amount was 120,000.

If we use a continuous growth equation, it would look like $N(t) = 120e^{rt}$, measured in thousands of users after *t* months. Based on the doubling time, there would be 240 thousand users after 5 months. This allows us to solve for the continuous growth rate:

$$240 = 120e^{r5}$$
$$2 = e^{r5}$$
$$\ln 2 = 5r$$
$$r = \frac{\ln 2}{5} \approx 0.1386$$

Now that we have an equation, $N(t) = 120e^{0.1386t}$, we can predict the number of users after 12 months:

$N(12) = 120e^{0.1386(12)} = 633.140$ thousand users.

So after 1 year, we would expect the site to have around 633,140 users.

Try it Now

3. If tuition at a college is increasing by 6.6% each year, how many years will it take for tuition to double?

Newton's Law of Cooling

When a hot object is left in surrounding air that is at a lower temperature, the object's temperature will decrease exponentially, leveling off towards the surrounding air temperature. This "leveling off" will correspond to a horizontal asymptote in the graph of the temperature function. Unless the room temperature is zero, this will correspond to a vertical shift of the generic exponential decay function.

Newton's Law of Cooling

The temperature of an object, T, in surrounding air with temperature T_s will behave according to the formula

$T(t) = ae^{kt} + T_s$

Where

t is time

a is a constant determined by the initial temperature of the object

k is a constant, the continuous rate of cooling of the object

While an equation of the form $T(t) = ab^t + T_s$ could be used, the continuous growth form is more common.

Example 7

A cheesecake is taken out of the oven with an ideal internal temperature of 165 degrees Fahrenheit, and is placed into a 35 degree refrigerator. After 10 minutes, the cheesecake has cooled to 150 degrees. If you must wait until the cheesecake has cooled to 70 degrees before you eat it, how long will you have to wait?

Since the surrounding air temperature in the refrigerator is 35 degrees, the cheesecake's temperature will decay exponentially towards 35, following the equation

$T(t) = ae^{kt} + 35$

We know the initial temperature was 165, so $T(0) = 165$. Substituting in these values,

$165 = ae^{k0} + 35$

$165 = a + 35$

$a = 130$

We were given another pair of data, $T(10) = 150$, which we can use to solve for k

$150 = 130e^{k10} + 35$

$115 = 130e^{k10}$

$\frac{115}{130} = e^{10k}$

$\ln\left(\frac{115}{130}\right) = 10k$

$k = \frac{\ln\left(\frac{115}{130}\right)}{10} = -0.0123$

Together this gives us the equation for cooling: $T(t) = 130e^{-0.0123t} + 35$.

Now we can solve for the time it will take for the temperature to cool to 70 degrees.

$$70 = 130e^{-0.0123t} + 35$$
$$35 = 130e^{-0.0123t}$$
$$\frac{35}{130} = e^{-0.0123t}$$
$$\ln\left(\frac{35}{130}\right) = -0.0123t$$
$$t = \frac{\ln\left(\frac{35}{130}\right)}{-0.0123} \approx 106.68$$

It will take about 107 minutes, or one hour and 47 minutes, for the cheesecake to cool. Of course, if you like your cheesecake served chilled, you'd have to wait a bit longer.

Try it Now

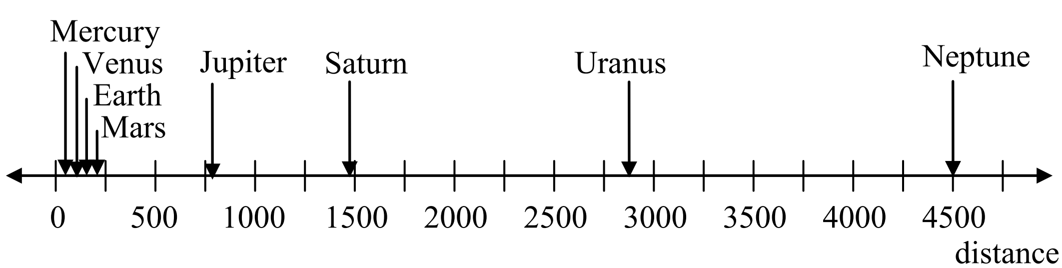
4. A pitcher of water at 40 degrees Fahrenheit is placed into a 70 degree room. One hour later the temperature has risen to 45 degrees. How long will it take for the temperature to rise to 60 degrees?

Logarithmic Scales

For quantities that vary greatly in magnitude, a standard scale of measurement is not always effective, and utilizing logarithms can make the values more manageable. For example, if the average distances from the sun to the major bodies in our solar system are listed, you see they vary greatly.

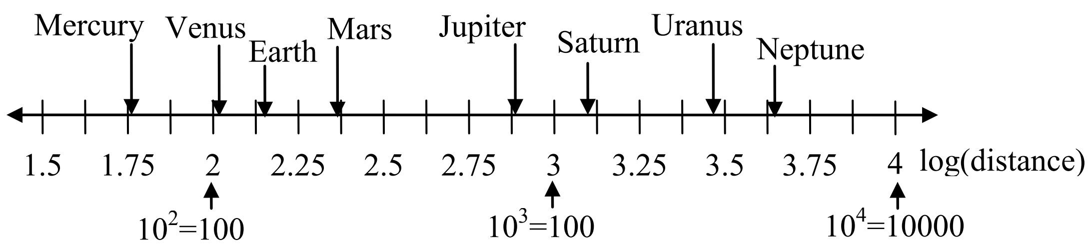
Planet	**Distance (millions of km)**
Mercury	58
Venus	108
Earth	150
Mars	228
Jupiter	779
Saturn	1430
Uranus	2880
Neptune	4500

Placed on a linear scale – one with equally spaced values – these values get bunched up.

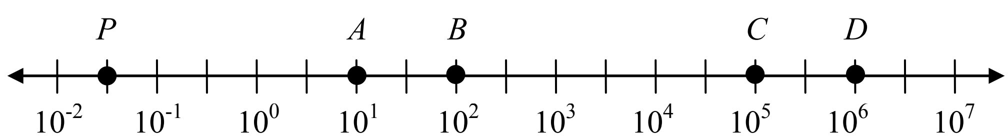


However, computing the logarithm of each value and plotting these new values on a number line results in a more manageable graph, and makes the relative distances more apparent.[4]

Planet	**Distance (millions of km)**	**log(distance)**
Mercury	58	1.76
Venus	108	2.03
Earth	150	2.18
Mars	228	2.36
Jupiter	779	2.89
Saturn	1430	3.16
Uranus	2880	3.46
Neptune	4500	3.65



Sometimes, as shown above, the scale on a logarithmic number line will show the log values, but more commonly the original values are listed as powers of 10, as shown below.



Example 8

Estimate the value of point P on the log scale above

The point P appears to be half way between -2 and -1 in log value, so if V is the value of this point,

$\log(V) \approx -1.5$ Rewriting in exponential form,

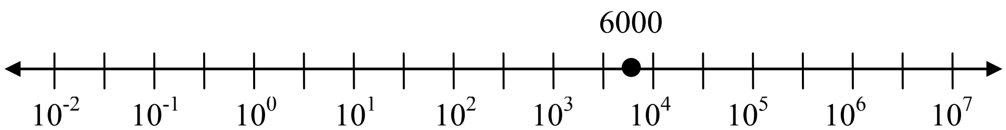
$V \approx 10^{-1.5} = 0.0316$

[4] It is interesting to note the large gap between Mars and Jupiter on the log number line. The asteroid belt, which scientists believe consists of the remnants of an ancient planet, is located there.

Example 9

Place the number 6000 on a logarithmic scale.

Since $\log(6000) \approx 3.8$, this point would belong on the log scale about here:



Try it Now

5. Plot the data in the table below on a logarithmic scale[5].

Source of Sound/Noise	Approximate Sound Pressure in µPa (micro Pascals)
Launching of the Space Shuttle	2,000,000,000
Full Symphony Orchestra	2,000,000
Diesel Freight Train at High Speed at 25 m	200,000
Normal Conversation	20,000
Soft Whispering at 2 m in Library	2,000
Unoccupied Broadcast Studio	200
Softest Sound a human can hear	20

Notice that on the log scale above Example 8, the visual distance on the scale between points *A* and *B* and between *C* and *D* is the same. When looking at the values these points correspond to, notice *B* is ten times the value of *A*, and *D* is ten times the value of *C*. A visual *linear* difference between points corresponds to a *relative* (ratio) change between the corresponding values.

Logarithms are useful for showing these relative changes. For example, comparing \$1,000,000 to \$10,000, the first is 100 times larger than the second.

$$\frac{1{,}000{,}000}{10{,}000} = 100 = 10^2$$

Likewise, comparing \$1000 to \$10, the first is 100 times larger than the second.

$$\frac{1{,}000}{10} = 100 = 10^2$$

When one quantity is roughly ten times larger than another, we say it is one **order of magnitude** larger. In both cases described above, the first number was two orders of magnitude larger than the second.

[5] From http://www.epd.gov.hk/epd/noise_education/web/ENG_EPD_HTML/m1/intro_5.html, retrieved Oct 2, 2010

Notice that the order of magnitude can be found as the common logarithm of the ratio of the quantities. On the log scale above, *B* is one order of magnitude larger than *A*, and *D* is one order of magnitude larger than *C*.

Orders of Magnitude

Given two values *A* and *B*, to determine how many **orders of magnitude** *A* is greater than *B*,

Difference in orders of magnitude = $\log\left(\dfrac{A}{B}\right)$

Example 10

On the log scale above Example 8, how many orders of magnitude larger is *C* than *B*?

The value *B* corresponds to $10^2 = 100$

The value *C* corresponds to $10^5 = 100{,}000$

The relative change is $\dfrac{100{,}000}{100} = 1000 = \dfrac{10^5}{10^2} = 10^3$. The log of this value is 3.

C is three orders of magnitude greater than *B*, which can be seen on the log scale by the visual difference between the points on the scale.

Try it Now

6. Using the table from Try it Now #5, what is the difference of order of magnitude between the softest sound a human can hear and the launching of the space shuttle?

An example of a logarithmic scale is the Moment Magnitude Scale (MMS) used for earthquakes. This scale is commonly and mistakenly called the Richter Scale, which was a very similar scale succeeded by the MMS.

Moment Magnitude Scale

For an earthquake with seismic moment *S*, a measurement of earth movement, the MMS value, or magnitude of the earthquake, is

$$M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

Where $S_0 = 10^{16}$ is a baseline measure for the seismic moment.

Example 11

If one earthquake has a MMS magnitude of 6.0, and another has a magnitude of 8.0, how much more powerful (in terms of earth movement) is the second earthquake?

Since the first earthquake has magnitude 6.0, we can find the amount of earth movement. The value of S_0 is not particularly relevant, so we will not replace it with its value.

$$6.0 = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$6.0\left(\frac{3}{2}\right) = \log\left(\frac{S}{S_0}\right)$$

$$9 = \log\left(\frac{S}{S_0}\right)$$

$$\frac{S}{S_0} = 10^9$$

$$S = 10^9 S_0$$

Doing the same with the second earthquake with a magnitude of 8.0,

$$8.0 = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$S = 10^{12} S_0$$

From this, we can see that this second value's earth movement is 1000 times as large as the first earthquake.

Example 12

One earthquake has magnitude of 3.0. If a second earthquake has twice as much earth movement as the first earthquake, find the magnitude of the second quake.

Since the first quake has magnitude 3.0,

$$3.0 = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$3.0\left(\frac{3}{2}\right) = \log\left(\frac{S}{S_0}\right)$$

$$4.5 = \log\left(\frac{S}{S_0}\right)$$

$$10^{4.5} = \frac{S}{S_0}$$

$$S = 10^{4.5} S_0$$

Since the second earthquake has twice as much earth movement, for the second quake, $S = 2 \cdot 10^{4.5} S_0$

Finding the magnitude,

$$M = \frac{2}{3}\log\left(\frac{2 \cdot 10^{4.5} S_0}{S_0}\right)$$

$$M = \frac{2}{3}\log\left(2 \cdot 10^{4.5}\right) \approx 3.201$$

The second earthquake with twice as much earth movement will have a magnitude of about 3.2.

In fact, using log properties, we could show that whenever the earth movement doubles, the magnitude will increase by about 0.201:

$$M = \frac{2}{3}\log\left(\frac{2S}{S_0}\right) = \frac{2}{3}\log\left(2 \cdot \frac{S}{S_0}\right)$$

$$M = \frac{2}{3}\left(\log(2) + \log\left(\frac{S}{S_0}\right)\right)$$

$$M = \frac{2}{3}\log(2) + \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

$$M = 0.201 + \frac{2}{3}\log\left(\frac{S}{S_0}\right)$$

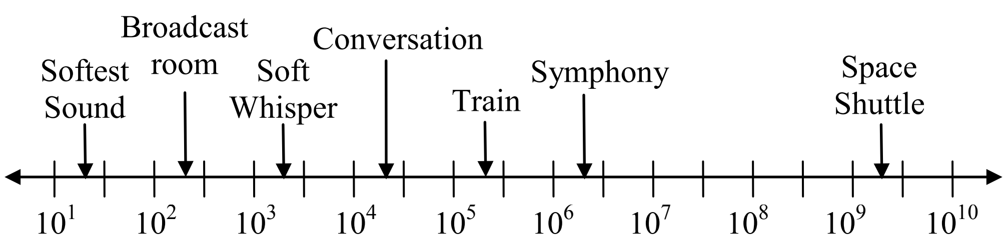
This illustrates the most important feature of a log scale: that *multiplying* the quantity being considered will *add* to the scale value, and vice versa.

Important Topics of this Section

Radioactive decay
Half life
Doubling time
Newton's law of cooling
Logarithmic Scales
Orders of Magnitude
Moment Magnitude scale

Try it Now Answers

1. $r = \sqrt[10]{\frac{1}{2}} - 1 \approx -0.067$ or 6.7% is the daily rate of decay.
2. Less than 230 years, 229.3157 to be exact
3. It will take 10.845 years, or approximately 11 years, for tuition to double.
4. 6.026 hours
5.



6. $\frac{2x10^9}{2x10^1} = 10^8$ The sound pressure in μPa created by launching the space shuttle is 8 orders of magnitude greater than the sound pressure in μPa created by the softest sound a human ear can hear.

Section 4.6 Exercises

1. You go to the doctor and he injects you with 13 milligrams of radioactive dye. After 12 minutes, 4.75 milligrams of dye remain in your system. To leave the doctor's office, you must pass through a radiation detector without sounding the alarm. If the detector will sound the alarm whenever more than 2 milligrams of the dye are in your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived and the amount of dye decays exponentially?

2. You take 200 milligrams of a headache medicine, and after 4 hours, 120 milligrams remain in your system. If the effects of the medicine wear off when less than 80 milligrams remain, when will you need to take a second dose, assuming the amount of medicine in your system decays exponentially?

3. The half-life of Radium-226 is 1590 years. If a sample initially contains 200 mg, how many milligrams will remain after 1000 years?

4. The half-life of Fermium-253 is 3 days. If a sample initially contains 100 mg, how many milligrams will remain after 1 week?

5. The half-life of Erbium-165 is 10.4 hours. After 24 hours a sample still contains 2 mg. What was the initial mass of the sample, and how much will remain after another 3 days?

6. The half-life of Nobelium-259 is 58 minutes. After 3 hours a sample still contains10 mg. What was the initial mass of the sample, and how much will remain after another 8 hours?

7. A scientist begins with 250 grams of a radioactive substance. After 225 minutes, the sample has decayed to 32 grams. Find the half-life of this substance.

8. A scientist begins with 20 grams of a radioactive substance. After 7 days, the sample has decayed to 17 grams. Find the half-life of this substance.

9. A wooden artifact from an archeological dig contains 60 percent of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years.)

10. A wooden artifact from an archeological dig contains 15 percent of the carbon-14 that is present in living trees. How long ago was the artifact made? (The half-life of carbon-14 is 5730 years.)

11. A bacteria culture initially contains 1500 bacteria and doubles in size every half hour. Find the size of the population after: a) 2 hours b) 100 minutes

12. A bacteria culture initially contains 2000 bacteria and doubles in size every half hour. Find the size of the population after: a) 3 hours b) 80 minutes

13. The count of bacteria in a culture was 800 after 10 minutes and 1800 after 40 minutes.
    a. What was the initial size of the culture?
    b. Find the doubling time.
    c. Find the population after 105 minutes.
    d. When will the population reach 11000?

14. The count of bacteria in a culture was 600 after 20 minutes and 2000 after 35 minutes.
    a. What was the initial size of the culture?
    b. Find the doubling time.
    c. Find the population after 170 minutes.
    d. When will the population reach 12000?

15. Find the time required for an investment to double in value if invested in an account paying 3% compounded quarterly.

16. Find the time required for an investment to double in value if invested in an account paying 4% compounded monthly

17. The number of crystals that have formed after t hours is given by $n(t) = 20e^{0.013t}$. How long does it take the number of crystals to double?

18. The number of building permits in Pasco t years after 1992 roughly followed the equation $n(t) = 400e^{0.143t}$. What is the doubling time?

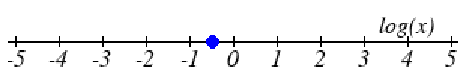
19. A turkey is pulled from the oven when the internal temperature is 165° Fahrenheit, and is allowed to cool in a 75° room. If the temperature of the turkey is 145° after half an hour,
    a. What will the temperature be after 50 minutes?
    b. How long will it take the turkey to cool to 110°?

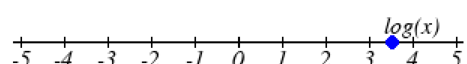
20. A cup of coffee is poured at 190° Fahrenheit, and is allowed to cool in a 70° room. If the temperature of the coffee is 170° after half an hour,
    a. What will the temperature be after 70 minutes?
    b. How long will it take the coffee to cool to 120°?

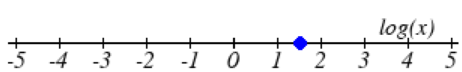
21. The population of fish in a farm-stocked lake after t years could be modeled by the equation $P(t)=\dfrac{1000}{1+9e^{-0.6t}}$.
    a. Sketch a graph of this equation.
    b. What is the initial population of fish?
    c. What will the population be after 2 years?
    d. How long will it take for the population to reach 900?

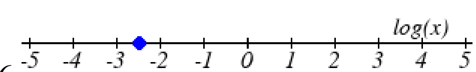
22. The number of people in a town who have heard a rumor after t days can be modeled by the equation $N(t)=\dfrac{500}{1+49e^{-0.7t}}$.
    a. Sketch a graph of this equation.
    b. How many people started the rumor?
    c. How many people have heard the rumor after 3 days?
    d. How long will it take until 300 people have heard the rumor?

Find the value of the number shown on each logarithmic scale

23. 

24. 

25. 

26. 

Plot each set of approximate values on a logarithmic scale.

27. Intensity of sounds: Whisper: $10^{-10}\ W/m^2$, Vacuum: $10^{-4} W/m^2$, Jet: $10^{2}\ W/m^2$

28. Mass: Amoeba: $10^{-5} g$, Human: $10^{5} g$, Statue of Liberty: $10^{8} g$

29. The 1906 San Francisco earthquake had a magnitude of 7.9 on the MMS scale. Later there was an earthquake with magnitude 4.7 that caused only minor damage. How many times more intense was the San Francisco earthquake than the second one?

30. The 1906 San Francisco earthquake had a magnitude of 7.9 on the MMS scale. Later there was an earthquake with magnitude 6.5 that caused less damage. How many times more intense was the San Francisco earthquake than the second one?

31. One earthquake has magnitude 3.9 on the MMS scale. If a second earthquake has 750 times as much energy as the first, find the magnitude of the second quake.

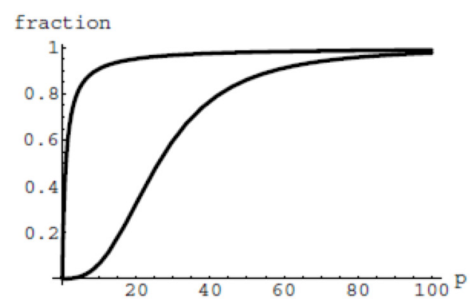
32. One earthquake has magnitude 4.8 on the MMS scale. If a second earthquake has 1200 times as much energy as the first, find the magnitude of the second quake.

33. A colony of yeast cells is estimated to contain 10^6 cells at time $t = 0$. After collecting experimental data in the lab, you decide that the total population of cells at time t hours is given by the function $f(t) = 10^6 e^{0.495105t}$. [UW]
    a. How many cells are present after one hour?
    b. How long does it take of the population to double?.
    c. Cherie, another member of your lab, looks at your notebook and says: "That formula is wrong, my calculations predict the formula for the number of yeast cells is given by the function. $f(t) = 10^6 (2.042727)^{0.693147t}$." Should you be worried by Cherie's remark?
    d. Anja, a third member of your lab working with the same yeast cells, took these two measurements: 7.246×10^6 cells after 4 hours; 16.504×10^6 cells after 6 hours. Should you be worried by Anja's results? If Anja's measurements are correct, does your model over estimate or under estimate the number of yeast cells at time t?

34. As light from the surface penetrates water, its intensity is diminished. In the clear waters of the Caribbean, the intensity is decreased by 15 percent for every 3 meters of depth. Thus, the intensity will have the form of a general exponential function. [UW]
    a. If the intensity of light at the water's surface is I_0, find a formula for $I(d)$, the intensity of light at a depth of d meters. Your formula should depend on I_0 and d.
    b. At what depth will the light intensity be decreased to 1% of its surface intensity?

35. Myoglobin and hemoglobin are oxygen-carrying molecules in the human body. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells. The function $Y = M(p) = \frac{p}{1+p}$ calculates the fraction of myoglobin saturated with oxygen at a given pressure p Torrs. For example, at a pressure of 1 Torr, $M(1) = 0.5$, which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the "partial pressure", but the distinction is not important for this problem.) Likewise, the function $Y = H(p) = \frac{p^{2.8}}{26^{2.8} + p^{2.8}}$ calculates the fraction of hemoglobin saturated with oxygen at a given pressure p. [UW]

a. The graphs of $M(p)$ and $H(p)$ are given here on the domain $0 \le p \le 100$; which is which?



b. If the pressure in the lungs is 100 Torrs, what is the level of oxygen saturation of the hemoglobin in the lungs?

c. The pressure in an active muscle is 20 Torrs. What is the level of oxygen saturation of myoglobin in an active muscle? What is the level of hemoglobin in an active muscle?

d. Define the efficiency of oxygen transport at a given pressure p to be $M(p) - H(p)$. What is the oxygen transport efficiency at 20 Torrs? At 40 Torrs? At 60 Torrs? Sketch the graph of $M(p) - H(p)$; are there conditions under which transport efficiency is maximized (explain)?

36. The length of some fish are modeled by a von Bertalanffy growth function. For Pacific halibut, this function has the form $L(t) = 200\left(1 - 0.957e^{-0.18t}\right)$ where $L(t)$ is the length (in centimeters) of a fish t years old. [UW]

a. What is the length of a newborn halibut at birth?

b. Use the formula to estimate the length of a 6–year–old halibut.

c. At what age would you expect the halibut to be 120 cm long?

d. What is the practical (physical) significance of the number 200 in the formula for $L(t)$?

37. A cancer cell lacks normal biological growth regulation and can divide continuously. Suppose a single mouse skin cell is cancerous and its mitotic cell cycle (the time for the cell to divide once) is 20 hours. The number of cells at time *t* grows according to an exponential model. [UW]
    a. Find a formula $C(t)$ for the number of cancerous skin cells after *t* hours.
    b. Assume a typical mouse skin cell is spherical of radius 50×10^{-4} cm. Find the combined volume of all cancerous skin cells after *t* hours. When will the volume of cancerous cells be 1 cm^3?

38. A ship embarked on a long voyage. At the start of the voyage, there were 500 ants in the cargo hold of the ship. One week into the voyage, there were 800 ants. Suppose the population of ants is an exponential function of time. [UW]
    a. How long did it take the population to double?
    b. How long did it take the population to triple?
    c. When were there be 10,000 ants on board?
    d. There also was an exponentially growing population of anteaters on board. At the start of the voyage there were 17 anteaters, and the population of anteaters doubled every 2.8 weeks. How long into the voyage were there 200 ants per anteater?

39. The populations of termites and spiders in a certain house are growing exponentially. The house contains 100 termites the day you move in. After 4 days, the house contains 200 termites. Three days after moving in, there are two times as many termites as spiders. Eight days after moving in, there were four times as many termites as spiders. How long (in days) does it take the population of spiders to triple? [UW]