Math 111
Activity for Unit 3

This activity will be an exploration of rational functions and their characteristics: vertical and horizontal intercepts, vertical and horizontal asymptotes and removable singularities (a.k.a. “holes”).

Part 1
To create a rational function that has horizontal intercepts of $(2, 0), (-3, 0)$ and vertical asymptotes of $x = 4$ and $x = -4$ we could define it as
$f(x) = \frac{(x-2)(x+3)}{(x-4)(x+4)}$.

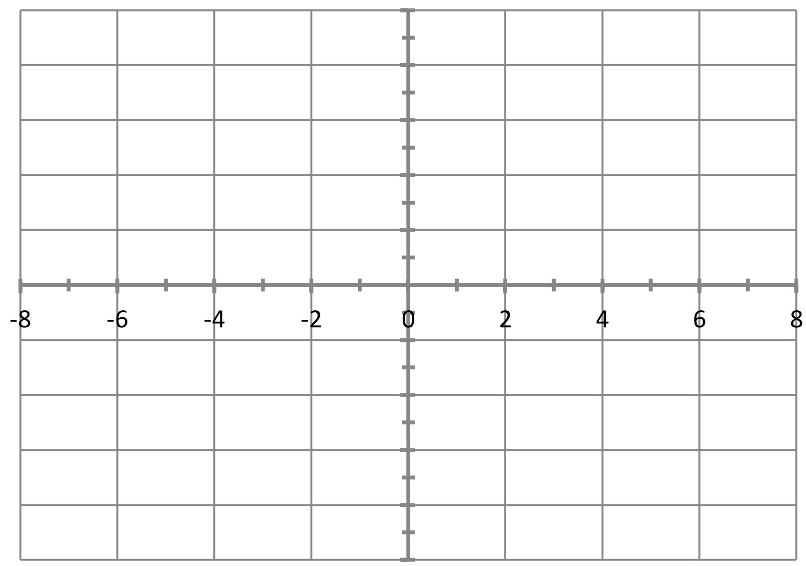
What will be the vertical intercept of your function $f(x)$?_____________

What would be the horizontal asymptote of $f(x)$? ______________

For your function $f(x)$ to have a vertical intercept at $(0, 5)$ you will need to do a little algebra and find a leading coefficient for the numerator. Take your function and insert an m as a placeholder for the leading coefficient:
$g(x) = \frac{m(x-2)(x+3)}{(x-4)(x+4)}$. Let $g(x) = 5$ and each $x = 0$ then solve for m.

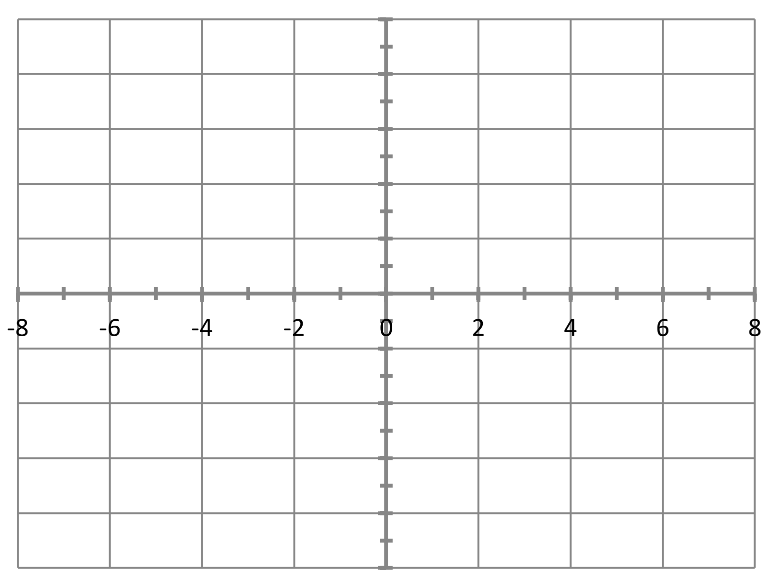
$g(x) =$

What will be the horizontal asymptote for your function? ________________
Now, graph your function showing all of the key characteristics. You will need to choose an appropriate scale for the vertical axis.

$g(x) =$ Horizontal intercepts: Vertical intercept: Horizontal asymptote: Vertical asymptotes:	

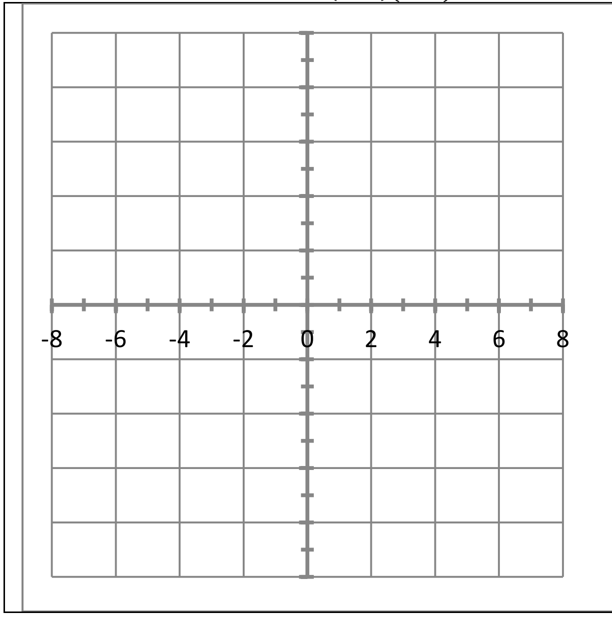
Part 2

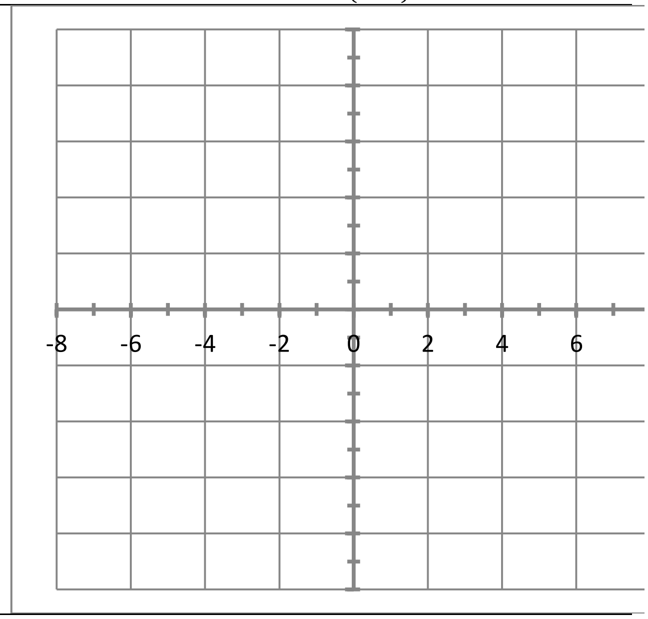
Now let's try some variations of your function. Go back to $f(x) = \frac{(x-2)(x+3)}{(x-4)(x+4)}$ but square one of your factors in the denominator: $h(x) = \frac{(x-2)(x+3)}{(x-4)^2(x+4)}$. Graph $h(x)$ below, and explain how it is similar and how it is different from the graph of $f(x)$.

	Explanation:

Part 3

Now we are going to make one of your vertical asymptotes disappear. Write a new function $j(x) = \frac{(x-2)(x+4)}{(x-4)(x+4)}$. Graph it below, as well as $k(x) = \frac{(x-2)}{(x-4)}$.





While at first glance the two graphs may appear the same, there is a critical distinction.

What is the value of $j(-4)$? ______________ What is the value of $k(-4)$? ________

The $j(x)$ function has a removable singularity at $x = -4$. Or, what is often called a "hole" at $x = -4$. Graphically, this is denoted by drawing an open circle on the graph at $x = -4$ to show that the function is not defined at that point.

Part 4

Last we will explore what happens to rational functions as input values approach a value not in the domain of the function. For each of the functions, complete the table below.

$l(x) = \dfrac{1}{(x+3)(x-2)}$	$m(x) = \dfrac{1}{(x+3)^2(x-2)}$	$n(x) = \dfrac{x+3}{(x+3)(x-2)}$

	Values left of $x = -3$			Values right of $x = -3$		
Function	-3.1	-3.01	-3.001	-2.9	-2.99	-2.999
$l(x)$						
$m(x)$						
$n(x)$						

For the function $l(x)$, what value does $l(x)$ approach as the values of x approach $x = -3$ from the left?

For the function $l(x)$, what value does $l(x)$ approach as the values of x approach $x = -3$ from the right?

In words, explain what happens to the graph of $l(x)$ at $x = 3$.

For the function $m(x)$, what value does $m(x)$ approach as the values of x approach $x = -3$ from the left?

For the function $m(x)$, what value does $m(x)$ approach as the values of x approach $x = -3$ from the right?

In words, explain what happens to the graph of $m(x)$ at $x = -3$.

For the function $n(x)$, what value does $n(x)$ approach as the values of x approach $x = -3$ from the left?

For the function $n(x)$, what value does $n(x)$ approach as the values of x approach $x = -3$ from the right?

In words, explain what happens to the graph of $n(x)$ at $x = -3$.